Name——————————————, **ID** ————————

Final Exam for MTH 221 , Spring 2011

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QUESTION 1. (12pts, each = 1.5 points) Answer the following as true or false: NO WORKING NEED BE SHOWN.

(i) If A is a 3×3 matrix and det(A) = 4, then det(3A) = 12.

(ii) If A is a 10×10 matrix and det(A) = 2, then $det(AA^T) = 1$

(iii) If Q, F are independent points in R^n, then $Q.F = 0$ (Q.F means dot product of Q with F).

(iv) $T(a, b, c) = (2ab, -c)$ is a linear transformation from R^3 to R^2.

(v) If A is a 3×3 matrix and $det(A - \alpha I_3) = (1-\alpha)^2(3+\alpha)$ and $E_1 = span\{(2, 4, 0)\}$, then it is possible that A is diagnolizable.

(vi) If $T : R^2 \to R^2$ is a linear transformation and $Ker(T) = \{(0, 0)\}$, then T is onto.

(vii) If A is a 4×5 matrix, then dimension of N(A) is at least one.

(viii) If A is a 3×4 matrix and Rank(A) = 3, then the columns of A are dependent.

QUESTION 2. (8pts) For what value(s) of k is the system of equations below inconsistent?

$$\begin{aligned} -x + y + z &= k \\ 2x - 3y + z &= 2 \\ -y + kz &= 6 + k \end{aligned}$$

QUESTION 3. (i) **(5pts)**For which value(s) of x is the following matrix singular (non-invertible)?

$$\begin{pmatrix} 1 & x & 2 \\ -1 & 1 & 1 \\ -1 & 5 & x+1 \end{pmatrix}$$

(ii) **(5pts)**Find examples of 2×2 matrices A and B such that

$$\det(A) = \det(B) = 2 \text{ and } \det(A+B) = 25,$$

or explain why no such matrices can exist.

QUESTION 4. (12pts) Let

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & -1 & 0 \\ 2 & -2 & 3 \end{pmatrix}$$

(i) Find A^{-1}.

(ii) Use your result in (i) above to solve the system

$$\begin{aligned} 2x - y &= 1 \\ x - y &= 2 \\ 2x - 2y + 3z &= 1 \end{aligned}$$

(iii) Solve the system $(A^T)^{-1}X = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

(If you need more space, then use the back of this page)

QUESTION 5. (12pts)

(i) Form a basis, say B, for P_4 such that B contains the two independent polynomials : $f(x) = 1 + x + 2x^2, k(x) = -2 - 2x + x^2$.

(ii) Let $S = span\{(1, 1, -1, 0), (0, 1, 1, 1), (3, 5, -1, 2)\}$. Find an orthogonal basis for S.

(iii) Let S be the subspace as in (ii). Is $(2, 5, 1, 3) \in S$? EXPLAIN your answer.

QUESTION 6. (12pts)

(i) Let $S = \{(a, bc + a, c) \mid a, b, c \in R\}$. Is S a subspace of R^3? If yes, then find a basis for S. If No, then tell me why not.

(ii) Let $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in R \quad and \quad a + b + c = 0 \right\}$. Is S a subspace of $R^{2\times 2}$? If yes, then find a basis for S. If No, then tell me why not.

(iii) Let $S = \{f(x) \in P_4 \mid f(1) = 0 \quad \text{OR} \quad f(-2) = 0\}$. Is S a subspace of P_4? If yes, then find a basis for S. If No, then tell me why not.

(iv) $S = \left\{ \begin{bmatrix} x & -x \\ 1 & y \end{bmatrix} : x, y \epsilon R \right\}$. Is S a subspace of $R^{2\times 2}$. If yes, then find a basis. If No, then tell me why not.

QUESTION 7. (14pts) Let $T : R^4 \to R^3$ such that $T(a, b, c, d) = (a + 2b,\ \ -a - 2b + c - d,\ \ -2a - 4b - c + d)$ be a linear transformation.

(i) **(3pts)**Find the standard matrix representation of T, say M.

(ii) **(4pts)**Find a basis for Ker(T).

(iii) **(4pts)**Find a basis for the range of T.

(iv) **(3pts)**Is $(-2, 1, 3, 3) \in Kert(T)$? Explain

QUESTION 8. (8 pts) Let $T : P_2 \to R^2$ be a linear transformation such that $T(1+x) = (-6, -2)$, and $T(2-x) = (-3, -1)$

(i) Find T(5) and T(3x)

(ii) Is there a polynomial $f(x) = a + bx$ such that $T(a + bx) = (6, 2)$? if yes, then find such $f(x)$

QUESTION 9. (12pts) Given $A = \begin{bmatrix} 1 & 4 & 4 & 4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ is a diagonalizable matrix.

(i) Find a diagonal matrix D and an invertible matrix Q such that $A = QDQ^{-1}$.

(ii) Find A^{2012}.

Faculty information

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